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Exploiting Structure: Introduction and Motivation

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1. Summary

This annual report briefly summarizes research activities performed during the period of June 26, 1993 through February 28, 1994. We continue to develop the Robust Stability of Systems where transfer function or characteristic polynomial are affine multilinear functions of parameters of interest in two directions, Algorithmic and Theoretical. In the algorithmic direction, a approach different form "Stability by Linear Process" that reduces the computational burden of checking the robust stability of the system with multilinear uncertainty was found for low order, 2-order and 3-order, cases. In the analysis, we proved a crucial theorem so-called Face Theorem as we have had the Kharitonov's Vertex Theorem, the Edge Theorem by Bartlett else. The detail of proof please see it in the Appendix. This Theorem provides a tool to describe the boundary of the image of the affine multilinear function. Thus, we will finally complete our paper. For SPR design, we develop some new results based on our work in semiannual report and the paper is in progress. The third work during this period is to design a controller for IHM by the H_∞ optimization technique. The details is shown in the Appendix.

2.1 Parametric Robust Stability

Let $p(s, Q) = \Sigma^{n}_{i=0} a_i(q)s^i$, $\quad q \in Q \subset R^n$

and $a_i(q)$ are affine multilinear functions. As we know that

$$p(s,Q) \in H \text{ if and only if } 0 \notin p(j\omega, Q)$$

Where H is Hurwitz space.

The objective of our research is to develop this criterion by algorithm and theoretical analysis.

In the algorithmic side, we gave a computational feasible algorithm so called "stability by linear process" to reduce the heavy computational burden. The speed of the algorithm can be determined by examining the best/worst case scenario. The average case requires $(n+1)/2$ iterations, and one iteration includes 5 steps calculations. The linear property in multilinear provides not enough informations to reduce a lot of computational burden and to prove our conjectures in our semiannual report.

According to the line of research of this field, we have had Kharitonov's Vertex Theorem (1978), the Edge Theorem by Bartlett, Hollot and Huang (1986) and they reduced the heavy computational burden for linear case. Logically, this motivated us to consider a Face Theorem Analytically. In our study, we found that the image of $F(Q) = p(j\omega, Q)$ is the union of the image of all possible 2-dimensional exposed edge of Q. There is a image of F(Q) for $n = 4$ shown in Appendix. Thus a crucial Face Theorem turns out.

Face Theorem for $Q \subset R^n$.

If F is a holomorphic function in C whose real part and imaginary part are affine multilinear functions of $Q \subset R^n$, and f_i, $i = 1, 2, \ldots, m$ are the two-dimensional exposed faces of Q with $m = C^2_n 2^{n-2}$, then $\cup^{m}_{i=1} F(f_i) = \text{Im } F(Q)$.

For this, we proved this theorem in the case $Q \subset R^3$ first, and then obtained the Face Theorem for $Q \subset R^n$. The details is shown in Appendix. This theorem will lead us to classify the boundary of F(Q) because we can get the information about F(Q) from the image of 2-dim exposed edges. That is to say that the study of the boundary of F(Q) becomes the study of the boundary of $F(f_i)$. The boundary of F(Q) could be classified as follows:

1. The boundary of F(Q) with 1-dim vertex as the ends is a line segment. The proof was done.
2. The boundary of F(Q) with 2-dim vertex as the ends is a parabola. The proof was done.
3. The boundary of F(Q) with 3-dim vertex as the ends is a cubic algebraic curve which is a set of points on the boundary of F(Q) with 2-dim vertex. The proof was done.
4. The boundary of F(Q) with k-dim vertex as the end is k-order algebraic curve. We have something else to be done.

Such fact provides the possibility to reduce Computational burden. We are doing now.

2.2 Strictly Positive Real Functions (SPR) for Robust Design

The basic problem in SPR is to determine if for a given Hurwitz Polynomial set Ω, there is a Hurwitz Polynomial d(s) such that

$$(n(s), d(s)) \in SPR \qquad \forall\, n(s) \in \Omega$$

and under which condition on Ω can be found to guarantee the existence of such a polynomial d(s).

In our 1993 semiannual report we completed the case for which Ω is a line segment. ie.,

$$\Omega = \{n(s) \in R^{n+1} : n(s) = \lambda n_1(s)+(1-\lambda)n_2(s) \text{ for } 0 \leq \lambda \leq 1,\ n_1(s), n_2(s) \in H\}.$$

We proved that there exist $d(s) \in H$ such that $\{\lambda\, n_1(s) + (1-\lambda)\, n_2(s), d(s)\} \in SPR$

if and only if $\lambda n_1(s) + (1-\lambda)n_2(s) \in H$.

In the proof, we apply its equivalent condition $SPR\{n_1(s)\} \cap SPR\{n_2(s)\} \neq \phi$.

In order to generalize the set Ω, we need to describe SPR{n(s)} based on the coefficients of n(s), in geometric terms.

Description of SPR{n(s)}.

Let $n(s) = \Sigma^n_{i=0} a_i s^i$ in H, and $n(s) = a_h(s^2) + s a_g(s^2)$

where $a_h(s^2) = a_0 + a_2 s^2 + a_4 s^4 + \ldots$, and $a_g(s^2) = a_1 + a_3 s^2 + a_5 s^4 + \ldots$

Then $SPR\{n(s)\} = B \cap I$.

where $B=\{(b_0, b_1, \cdots, b_n) \in R^{n+1} : \Sigma^n_{i\in E} a_h(-\omega^2)\omega^i b_i + a_g(-\omega^2)\omega^i b_{i+1} > 0,$
$\forall \omega \in R$, E is even integers$\}$

and $I = \{(b_0, b_1, \cdots, b_n) \in R^{n+1} : b_i \geq 0 \text{ for } i=0,1,2,\cdots,n\}$

is the first octant of the coefficient space.

It is evidence that SPR{n(s)} is a convex cone in R^{n+1}.

The Generalizations

1. For the case in which Ω is a Kharitonov Box, we have that there exist $d(s) \in H$ such that

$\{\Sigma^8_{i=1} \lambda_i\, n_i(s), d(s)\} \in SPR$, and $\Sigma^8_{i=1} \lambda_i = 1$, for the vertex polynomial $n_i(s) \in H$

if and only if $\Sigma^{8}_{i=1} \lambda_i \, n_i(s) \epsilon H$.

For this, it is sufficient to show that

$$\cap^{8}_{i=1} \text{SPR}\ (n_i(s)) \neq \phi$$

Indeed, this is equivalent to the existence of a solution to the system of inequalities

$$\Sigma^{n}_{i\epsilon E}\ a^{1}_{h}(-\omega^2)\omega^{i}b_i + a^{1}_{g}(-\omega^2)\omega^{i}b_{i+1} > 0$$

$$\Sigma^{n}_{i\epsilon E}\ a^{2}_{h}(-\omega^2)\omega^{i}b_i + a^{2}_{g}(-\omega^2)\omega^{i}b_{i+1} > 0$$

$$\vdots$$

$$\Sigma^{n}_{i\epsilon E}\ a^{8}_{h}(-\omega^2)\omega^{i}b_i + a^{8}_{g}(-\omega^2)\omega^{i}b_{i+1} > 0, \text{ and}$$

$$b_i > 0 \text{ for } i = 0, 1, \cdots, n.$$

This solution does exist.

2. Consider the case in which Ω is a polytope. If V represents the set of vertices of Ω, we have that there exist $d(s) \epsilon H$ such that

$\{\Sigma_{i\epsilon V}\ \lambda_i n_i(s),\ d(s)\}\epsilon$SPR, where $\Sigma_{i\epsilon V}\lambda_i = 1$, for the vertex polynomial $n_i(s)\epsilon H$

if and only if

$$\Sigma_{i\epsilon V}\ \lambda_i \, n_i(s) \epsilon H.$$

For this, we proved that $\cap_{i\epsilon V}$ SPR $(n_i(s) \neq \phi$

This is equivalent to the existence of a solution to the system of inequalities

$$\Sigma^{n}_{i\epsilon E}\ a^{1}_{h}(-\omega^2)\omega^{i}b_i + a^{1}_{g}(-\omega^2)\omega^{i}b_{i+1} > 0$$

$$\Sigma^{n}_{i\epsilon E}\ a^{2}_{h}(-\omega^2)\omega^{i}b_i + a^{2}_{g}(-\omega^2)\omega^{i}b_{i+1} > 0$$

$$\vdots$$

$$\Sigma^{n}_{i\epsilon E}\ a^{v}_{h}(-\omega^2)\omega^{i}b_i + a^{v}_{g}(-\omega^2)\omega^{i}b_{i+1} > 0, \text{ and}$$

$$b_i > 0, \text{ for } i = 0, 1, \cdots, n$$

and this system has a solution.

3. For more generalizations of Ω, we have something else to be done.

The significance of this work is for proving the convergence of some algorithms in the identification of systems and designing a transfer function which is SPR invariant in the parameter identification problem.

2.3 ACS

2.3.1 The Robust Stability and Robust Performance of the Inertial Hold Mode (IHM)

In accordance with the requirement from Mr. Mosier, the main objective of this research is to design a controller for the Inertial Hold Mode (IHM). The controller will stabilize the plant with uncertainties from the natural frequencies of the flexible body, and the very weak structural damping value that results in hormonic vibration of the spacecraft. Also, it must perform as a good regulator to provide an adequate rejection to the torque disturbance acting on the spacecraft.

In the IHM, we are facing the control of a systems with flexible body having parametrically defined uncertainty in the natural frequencies ω_i and the external torque disturbances. It is well-known that the H_∞ design methodology is tailor-made for handling ignored high frequency dynamics and thus is ideally suited for the control of flexible structures. However, this does not automatically guarantee robustness to the low frequency parametric uncertainty in IHM. The problem is how to develop techniques for modeling the parametric uncertainty so that:

- The H_∞ design technique can be used.
- The level of robust performance is maximized.

Our main work is to modify IHM into a typical Mixed-Sensitivity Problem in H_∞ and to design a controller K(s) such that the nominal system, G(s), is stable and such that

$$\{\| W_1 (1 + GK)^{-1} \|_\infty + \| W_3 GK(1 + GK)^{-1} \|_\infty\} < 1.$$

In this work, our main contributions are as follows:

(1) Modeling the parametric uncertainty with a single non-parametric block and retaining the simplicity and intuition of single-loop control. That is, we give a non-parametric maximum additive perturbation $\Delta_a(s)$ to cover the entire flexible body as an additive perturbation, and then turn out a multiplicative perturbation $\Delta_m(s)$ for convenience making the nominal system, G(s), and the adequate weighting function, $W_3(s)$, for maximizing the robust stability.

(2) Defining the weighting function $W_1(s)$ for maximizing the robust performance and getting an adequate rejection to torque disturbance.

(3) In order to employ the H_∞ optimization algorithm directly, we found a suitable transformation of the regulator to shift the imaginary axis.

(4) In order to obtain good robust stability and robust performance, we adjust W_1 and W_3 alternatively to simulate the controller.

(5) To reduce the order of the controller, we eliminate those zeros and poles of the controller that are far away from the imaginary axis or the origin. This is for maintaining robustness (stability and performance) of the system, ie., maintaining a good performance of the open-loop transfer function of the system with reduced controller in the low and middle frequency bandwidths.

Finally, let us make a comparison between the simulation results of the H_∞ design and the PID design.

	H_∞ Design	PID Design
(a) Gain Margins	< -24.04db and > 24.04db	< -21db and > 14db
(b) Phase Margins	> 55.03°	> 45°
(c) Robustness with parametric uncertainty +/- 25%	Good	Not good
(d) Attenuation	150 Seconds	> 200 Seconds
(e) Controller's Order	≥ 4	2

2.3.2 The Controller Reduction

The controller in IHM is designed and satisfied to the specific requirements but its order is too high.

In general, the order of the controller designed by the H_∞ technique is guaranteed to be not higher than that of the generalized plant. However, in many engineering problems, the order of generalized plant could be very high. This is due to the fact that the generalized plant consists of the original plant as well as all the weighting matrices which are chosen to meet certain design specification. Hence, the order of the H_∞ controller obtained by the standard State-Space approach is usually too high to be implemented in practice, and therefore a systematical methodology of designing a reduced-order H_∞ robust controller is desired.

According to our investigation of the literature, several existing reduction methods have large error. The Balanced Stochastic Truncation (BST) method is good in high frequency bandwidths but is bad in the low and middle frequency bandwidths. We develop such reduction method for which the open-loop frequency characteristic of the system with reduced controller approaches to the original as close as possible and the stability and performance are retained perfectly. In terms of a formula,

$$\| KG - K_rG \|_\infty = \| (K - K_r)G \|_\infty$$

is as small as possible, where K_r is a reduced controller, and the generalized plant G(s) becomes a frequency weighted function in practice. The Frequency Weighted Balanced Truncation method by Ends could be used. The details in the paper for controller reduction is in progress and will be submitted to CDC of IEEE by March 1, 1994.

Appendix

1. Image of F(Q) for $Q \subset R^4$
2. The Face Theorem
3. H_∞ Optimal Controller Design and Reduction for IHM